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An Electronic Warfare Decision Aiding System for Fleet Air **Defense: An R&D Status Report**

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Prepared for:

Office of Naval Research 800 North Quincy Street Arlington, VA 22217

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AN ELECTRONIC WARFARE DECISION AIDING SYSTEM
FOR FLEET AIR DEFENSE:
AN R&D STATUS REPORT

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FOR FLEET AIR DEFENSE:

AN R&D STATUS REPORT

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The conceptual design for an electronic warfare emissions cont					
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was developed as a prototype to demonstrate some of the concepts. A brief description of the demonstration prototype is included and further steps required to develop and test a more complete prototype system are discussed.

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EXECUTIVE SUMMARY

This study was supported by the Office of Naval Research as a part of a larger project concerned with the development of automated and semiautomated decision aids to assist the commander and staff of a Naval Task Force. Previous GRC analysis (Mathematical Decision Aids for the Task Force Commander and His Staff, AD #A 023 940, GRC 593-01-CR) had identified the problem of fleet air defense as a particularly promising area for the development of mathematical decision aids. At the beginning of this project, after a general review of fleet air defense to identify specific problems most likely to benefit from mathematical decision aids, the area of electronic warfare was chosen for this study. It was chosen not only because it tends to be critical to combat outcomes but also because of the large quantity of technical data and the complex system interactions that make it a difficult area for the application of military judgment. For these reasons, it was concluded that analysis tools to assist the planner in assessing the effects of electronic policy on potential military outcomes might contribute substantially to improved decisions.

Specifically, emission control policy was selected for detailed analysis, and the effort was directed toward the development of a conceptual design for an EP (emission policy) decision aiding system. The resulting design includes three main components:

- (1) An air surveillance evaluation component, which provides a quantitative estimate of the air surveillance performance that can be expected under alternative emission control plans.
- (2) An information analysis system, which provides a quantitative assessment of the information concerning the identity of ships (particularly high-value ships) that can be given away to the enemy as a consequence of radar and other electronic emissions under any specified emission control plan.

(3) Two decision aiding displays to assist the user in making trade-offs between surveillance performance and information given away by his own ship's emissions. One of these simply displays a combined judgmental performance score, as a function of the judgmental weights assigned to the two types of information. The second is based on a quantitative calculation of overall air defense effectiveness, which is estimated as a function of both the air surveillance coverage and the information given away to the opponent (which can be exploited to plan more effective air strikes against the task force).

During the course of the study an initial prototype version of the air surveillance portion of this decision aiding system was developed and demonstrated.

To provide a realistic evaluation of the overall decision aiding concept, the prototype development should be extended to include the complete system, and the system should be tested in a simulated fleet environment to evaluate the potential usefulness of the concept and to compare alternative wasy of displaying the information available in the system.

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1 INTRODUCTION

This study was supported by the Office of Naval Research as a part of a larger porject concerned with the development of automated and semiautomated decision aids to assist the commander and staff of a Naval Task Force. Previous analysis by General Research Corporation (GRC) (Mathematical Decision Aids for the Task Force Commander and His Staff, AD #A 023 940, GRC 593-01-CR) had identified the problem of fleet air defense as a particularly promising area for the development of mathematical decision aids. Consequently the initial effort under this contract was concerned with the identification of those specific areas in fleet air defense that would benefit most from mathematical decision aids. A number of initial visits were made by GRC personnel to discuss the problems of fleet air defense with experts at Johns Hopkins Applied Physics Laboratory, the Naval Research Laboratory, the Naval War College at Newport, Rhode Island, the Tactical Action Officer Training School at Dam Neck, Virginia, and a number of Naval officers (at the Pentagon) with air defense experience. These discussions revealed that some of the most difficult issues in fleet air defense seemed to be associated with the electronic warfare aspects of the problem. Indeed, the electronic warfare problems seemed to provide a particularly promising area for aids to human planning because of the large quantity of technical data involved, and the obvious difficulties in visualizing the interactions of electronic systems. Many other air defense problems could, of course, have been addressed, but it appeared on balance that a decision aiding system dealing with electronic warfare issues might be the most useful. Moreover, because a study group at Stanford Research Institute was specifically working on a combined air strike-air defense planning aid, the choice of the electronic warfare area for the GRC project would tend to minimize overlap between the two studies.

How to balance the assignment of aircraft between air strike and fleet air defense activities; where to station combat air patrol aircraft and what aircraft to assign; how to coordinate the activities of manned interceptors with missile defense ships; etc.

With the approval of the ONR project monitor, the GRC effort was focused specifically on electronic warfare (EW). A new round of discussions was held with electronic warfare experts at NAVELEX, the Naval Research Laboratory, the Second Fleet, the Johns Hopkins Applied Physics Laboratory, and the training school at Dam Neck. Those discussions led to the decision to concentrate the effort primarily on the issue of emission control policy, and they provided the background for the overall design concept that is developed in this report.

OVERVIEW OF THE EMISSIONS POLICY PROBLEM

Modern naval warfare is largely a battle of information. Success or failure is critically dependent on the accuracy and reliability of information available to commanders on both sides. Moreover, the tactics of cover and deception are likely to play a central role in any future naval battle. Because modern electronic and acoustic detection systems provide some of the most important sources of combat information, plans for cover and deception usually require close coordination with emission control policy.

Present EW plans are usually developed in a rather perfunctory or standardized way that does not really tailor them to the circumstances, or allow them to be adapted dynamically to the objectives of a particular mission. A well-designed emissions policy decision aiding system should not only help the Task Force EW officer develop plans that are better tailored to the particular mission environment, but it should also provide displays that will allow the commanding officer to evaluate more accurately the key choices that are available for the control of his emitters.

Emissions policy is concerned primarily with the acquisition and denial of information. The objective of any emission control plan is to provide the most favorable possible balance of information, specifically, to ensure that the Task Force is able to obtain the essential information needed for its mission and to deny information to the enemy or to mislead the enemy by providing him with misleading information.

Present planning procedures call for the development of what is called an EMCON (emissions control) plan as part of the Task Force OP Order. The original concept of the EMCON plan was to specify appropriate restirctions on the operation of any emitters (i.e., radar, sonar, or communcations equipment) that could give important information to the enemy. More recently, especially in the Second Fleet, this perspective has been reversed, and the policy has been to avoid any emissions unless they are really important to the Task Force

mission. With this change in perspective, the plans in the Second Fleet are now described as EMREL (emission release) plans. However, NATO procedures are still based on the EMCON concept.

Actually, of course, a trade-off is always involved. Restricting the use of one's own electronic equipment imposes restrictions on certain Task Force capabilities and functions. The question is whether the denial of information to the enemy is of greater importance. The objective of an EMCON or EMREL planning aid should be to provide a better understanding of the information exchange involved so that better trade-off decisions can be made.

At the present time, the need for an emissions policy planning aid tends to be obscured by severe operational difficulties that are experienced in implementing existing EMCON plans. Experience in exercises has demonstrated that as a practical matter the commanding officer does not really have control over the ship's emissions. Without positive central control of emitters, some may be inadvertently left on or restored in an untimely manner.

In addition to the operational problems, there are also some rather serious technical problems that uncessarily degrade readiness. Electronic equipment cannot be maintained in a high state of readiness unless it is operated at least occasionally. To allow the equipment to be turned on for maintenance (or to be kept in a ready status despite EMCON requirements), many radar systems have been provided with a dummy load which theoretically should allow the equipment to operate without actually radiating. Unfortunately, in many cases the existing dummy load configurations are technically defective so that there is substantial radiation even in the dummy load configuration. These technical and operational problems have been so severe that recent priorities in the EW community have been focused on the implementation rather than the planning aspect of EMCON.

Implementing EMCON plans can be traced to three fundamental problems:

- (1) The commanding officer does not have any central control panel that will give him positive control over the radiation of the electronic equipment of his ship. He depends upon individual members of the crew to observe the EMCON regulations.
- (2) There is no comprehensive capability to monitor emissions from the ship, or from other ships in the Task Force, so for many pieces of equipment there may be no way of knowing that they have been left on.
- (3) Even though existing EMCON plans tend to be rather routine, they are not sufficiently standardized to allow crews to memorize procedures for each plan. Consequently, implementation of a plan requires a crew member to check each piece of equipment against the proper EMCON Op Order. Even after they check the Op Order, crews may be slow and error prone in implementing the plan.

It is obvious that EMCON plans cannot be effectively implemented unless the commanding officer is provided with a positive control capability and an effective capability to monitor the ships in the Task Force for EMCON compliance.

The Navy now has under development a positive control system called MUTE, which will allow central control over each ship's emissions. Although this system should give the captain of each ship positive control over his own ship's emissions, it will not provide either positive control or a monitoring capability at the Task Force level. Most EW officers now seem to be convinced that MUTE will solve the operational problems. However, this view may be overly optimistic. Although MUTE will obviously help, it seems likely that the problems will not really be resolved until the Task Force EW officer is provided with an effective monitoring capability. A rather typical incident will explain the problem. On a recent training exercise, the EW officer reported that one of the ships in the Task Force was oper-

ating the Mk-_ radar in violation of the EMCON plan. However, at that frequency, he had no direction finding capability and therefore could not identify the offending ship.

Because of the current difficulties in the implementation of EMCON plans, there are wide differences of opinion about what changes in EMCON planning procedures might be most helpful. Some operational officers will argue that the real problem is the lack of standarization of plans. Each EW officer produces different plans. If the fleet would only settle on a few standardized plans, they believe the operational problems could be solved. EW specialists, however, are more likely to believe that the present use of almost routine plans that are not tailored to the specific situation is the most serious problem.

An objective appraisal of the situation would probably conclude that the lack of an effective capability to implement EMCON plans is, at present, the most serious immediate problem. Although the implementation problems appear at present to be the highest priority issue, they do not fall within the responsibility of the ONR decision aiding project. After these problems have been resolved, however, emphasis seems likely to shift to developing really effective plans that can be tailored to a particular situation. The EW decision aiding concept that is developed here is aimed at such a time period.

3 GENERAL DESIGN APPROACH

The present EMCON planning procedures are sufficiently primitive that even the first steps of organizing the data and providing the EW officer with an effective data retrieval capability would be a major advance. At present, the EW officer can be so preoccupied with the problems of obtaining current EW information to use in the planning process that the planning and decision process itself gets very little attention. If an EW decision aid is to meet practical operational needs and gain wide acceptability, it will be necessary to provide capabilities that range from very simple data retrieval assistance to more sophisticated decision aid functions. The design concept for the decision aiding system that will be considered here has been developed in accordance with this principle.

The present development of an EW plan begins with the collection of a large amount of information. Roughly speaking, the steps are as follows:

- Identify the specific ships and types of aircraft that are relevant to the problem:
 - a. Own forces
 - Enemy forces
 - c. Neutral forces
- 2. For each platform (ship or aircraft), identify relevant electronic equipment:
 - a. Emitters (radar, sonar, communication transmitters)
 - b. Sensors (ESM, passive sonars, radio receivers)
 - c. Jammers
- 3. For each equipment, obtain electronic characteristics.
- 4. Match up equipment by frequency range.
- 5. For each anticipated EMCON option, evaluate for each equipment the advantages and disadvantages of having the equipment in operation.

ESM, electronic warfare support measures refers generally to equipment designed to monitor portions of the electronic spectrum.

6. Decide equipment status for each EMCON option.

The steps in this process that are most relevant from the perspective of the decision aids project are, of course, steps 5 and 6. However, these cannot be carried out until the preceding four steps have been completed. The decision aid concept described here is intended to provide assistance with all of the steps in the process.

Although the simple data retrieval and data analysis functions that will be discussed first may not seem to directly address the decision aiding function, they are a necessary precondition for that function. Within the overall design concept, these preliminary steps serve two separate purposes. First, they meet the critical need of EW specialists for such basic data management support. Second, they provide the foundation of data processing capabilities that is need to support an EW decision aiding system.

3.1 BASIC DATA AND DATA PROCESSING REQUIREMENTS

Most of the necessary EW information is already included in prototype form as a part of the ONR decision aids data base. Substantial additional information will probably not be required, although during the development of the decision aid certain oversights may be detected in the EW information that is currently planned.

The data processing capabilities already provided by the University of Pennsylvania for the ONR project appear to be well suited to the requirements of the EW decision aid. Probably, the easiest way to describe these requirements is to discuss the way a data management system would be used in the planning process.

The first step in the planning process is to identify the ships and aircraft, friendly, hostile, and neutral, that are likely to be relevant to the problem. The user will want to begin by supplying the system with this basic information on the forces involved. In the case of his own forces, he will know exactly what units are involved. In the case of the enemy forces, there may be some uncertainty, so it may be desirable to

allow the user to specify an estimate of the probability of involvement for specific platforms or types of platforms. At the request of the user, the system should then be able to retrieve and display the specific electronic equipment for each platform. Similarly, it should be able to retrieve the electronic characteristics information for each piece of equipment. The existing University of Pennsylvania "relational" data base seems ideally suited for this task.

3.2 VIDEO DISPLAY CAPABILITY

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If the system did nothing but retrieve and tabulate such information, however, the user would be presented with a massive quantity of undigested information. Consequently, the information must be organized and presented in a way that is most useful for the EW planner. Fortunately, the EW problem lends itself quite easily to display in frequency space. Experienced EW officers often expend a great deal of effort on maintaining personal notebooks that organize the information graphically in frequency space. Thus, there should be no doubt about the usefulness of such a frequency display.

The display might take a left-to-right form in frequency space as shown in Fig. 1, using color-coded horizontal bars to define the frequency range for each piece of equipment. The actual assigned frequency for each piece of equipment could be shown as a heavy line with a light or dotted line showing the frequency capabilities. The bars for different pieces of equipment could be staggered vertically to avoid overlap in the display. The screen should probably be divided into three sections. For example, the upper third could be devoted to friendly equipment, the middle third to hostile, and the lower third to neutral (including both neutral ships and commercial communication channels in the environment). The user should be able to specify a frequency range of interest and obtain a complete plot (probably in the logarithm of frequency) of all relevant equipment in the specified frequency range. Color codes might be used to identify types of equipment—for example, red for radar, orange for jammers, green for ESM, yellow for communications. The frequency

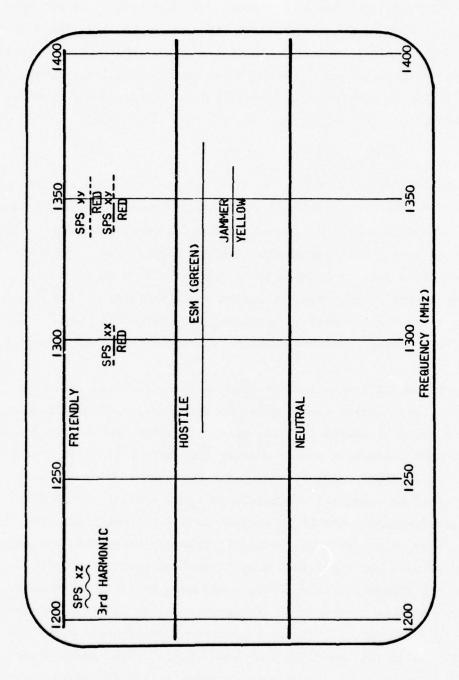


Figure 1. Illustration of EW Display in Frequency Space

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display for the radars and other major emitters should include not only the fundamental frequency but also the major frequency harmonics that are likely to cause interference with ESM and communications equipment.

This type of display has many potential uses:

- To plan the allocation of friendly ESM coverage of the frequency bands.
- 2. To allocate the actual frequencies to U.S. fire control and surveillance radars to minimize mutual interference.
- To help evaluate the vulnerability of friendly emitters to enemy ESM.
- 4. To check for possible peacetime interference with commercial channels.

This type of frequency display should be designed for use in an interactive mode. The user should be able to specify what equipment is to be silent, change frequency assignments for others, and then obtain a new display. In this way, the user will be able to visually inspect a display in frequency space to detect errors or oversights in his plan.

To simplify the specification of EMCON plans, the user should be able to specify broadly the general structure of the plan and then specify exceptions in detail. To facilitate such an approach, a hierarchy of levels of detail is needed in which the more detailed specifications override the less detailed. For example, the user might first specify total silence in bands E through H. He would then specify any exceptions in terms of specific types of equipment. Finally, he might specify exceptions within an equipment type for particular platforms.

This type of video display would provide the EW officer with automated assistance that would bypass a large fraction of the tedious work that is now involved in the development of EMCON plans.

The display is, in fact, a very rudimentary form of outcome calculator. For any specified EW plan, the system calculates an "outcome" in frequency space and displays the outcome for review by the decision maker.

3.3 DATA RETRIEVAL REQUIREMENTS FOR ESM THREAT IDENTIFICATION

The basic data retrieval functions for such a video display involve a capability to start with the specified platforms, and retrieve the associated electronic equipment and the electronic characteristics of the equipment. Many other ESM activities, however, require a capability to reverse this retrieval process. Having detected a particular electronic emission which can be characterized by frequency, pulse length, repetition rate, etc., the type of equipment needs to be identified and then the types of platforms that could be carriers of the equipment need to be determined. Fortunately, the relational data base also provides a very effective capability for such reverse retrieval.

The general ESM threat identification problem is somewhat more complex. In general, a signal can be ambiguous so that it could be attributed to more than one possible type of emitter. In addition, ESM surveillance will often identify more than one emitter on a single platform. Thus, a kind of Boolean logic is needed to identify the types of platforms that could be responsible for both emissions. This type of logic should undoubtedly be included as a part of the ESM decision aid.

To use such a system, the user would input the characteristics of emissions observed from a platform and as output would obtain a list of possible platforms. For the system to be useful as possible, the user should be able to specify whether the analysis is to be done only with respect to platforms he has specified as relevant, or whether it is to done for all platforms in the data base.

Although such an ESM threat identification capability has been described so far as an operational capability (for use when ESM signals are actually received) the same capability is required in the development of EMCON plans. One of the key questions that the EW planner must consider is the degree to which his emitters will give away identity information, especially for his high-value surface ships. For any specified assumption about which emitters are turned on and detected by enemy ESM, the same analysis system could be used to assess how much identity information might be derived by enemy intelligence. Specifically, the system could provide a report indicating what other ships could generate the same pattern of electromagnetic emissions.

3.4 TRADE-OFF SUMMARY

The essential problem that must be faced by the EW planner concerns the balance of information. How should the Task Force be configured electronically so that the balance of information will be as favorable as possible? Turning off radars and emitters denies certain information that otherwise might be acquired by enemy ESM equipment. On the other hand this also denies to our own forces information that might otherwise be obtained through active use of the emitters. In some cases, the development of deception plans will also involve the use of simulated emissions to mislead enemy ESM analysts. One of the obvious function of a decision aid should be to help the planner evaluate the specific information involved in such trade-offs.

The best emissions policy for any specific operation depends not only on the actual information balance but also on the importance of specific information to the Task Force objective. Consequently, to arrive at good decisions, it will be necessary for the planner to supply value judgments about the relative importance of the various kinds of information. Thus, such a decision aid should operate in partnership with the planner. The decision aid would help in identifying the specific information involved, and the planner would evaluate the tactical importance of the information.

Conversations with Naval Officers have revealed some interesting differences in perspective with regard to this information trade-off. Very few officers now see the problem intuitively as a trade-off. Officers with an air defense orientation seem to think of maximizing air defense capabilities subject to EMCON constraints, whereas the EW officers tend to think minimizing emissions that could give away information subject to the requirement to maintian some essential level of air defense capability. One of the important effects of an EMCON decision aid might be to encourage the planners to view the problem from the broader perspective of an information trade-off.

To provide such a capability, three basic analysis components are needed:

- An emissions analysis tool that can be used to assess the information that may be given away by having various combinations of equipment turned on, and the confusion that can be introduced through deception (i.e., the use of simulated radiators).
- 2. An <u>air defense radar surveillance model</u> that can be used to assess the effects of various EMCON restrictions on air defense capabilities, both in an unjammed and a jamming environment.
- 3. Some procedures to assist the user in making the value judgments that are needed to balance these two considerations.

In actual EMCON planning, there are many other considerations which must be taken into account such as: the effects of communication restrictions on Task Force performance; and the policy to be followed with regard to fire control radars and the selected use of jammers to degrade enemy capbabilities. However, the foregoing three components provide a basic capability to deal with some of the most important trade-offs, and they define a logical framework within which user judgment can be applied to take still other factors into account. Moreover, if the basic concept proves successful, the framework could easily be expanded to include some of the related analytical capabilities.

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Thus, in terms of the present ONR objective of providing a demonstration prototype, the preceding components seem to define a satisfactory point of departure. The next three sections summarize the proposed design concept for each of these subsystems.

4 THE EMISSIONS ANALYSIS DECISION AID

When electronic emissions are intercepted by enemy ESM equipment, they can provide the enemy with three different types of information:

- 1. Detection of presence
- 2. Location of individual units
- 3. Identification of individual units

The proposed emissions analysis aid will focus primarily on the problem of identification. Such identification information is important because it allows the enemy to distinguish and concentrate his resources of highvalue components of the Task Force such as the carriers and cruisers. Although the other two types of information are also important, there does not appear to be as much need for an analysis system to deal with them.

4.1 DETECTION

The assessment of ESM detection is relatively straightforward. Current trends in ESM equipment suggest that, in the future, any radar that is within the frequency coverage of ESM gear should be detected almost as soon as it comes on the air, so long as it is within the radar horizon. Thus, the calculation of detection probability can be accomplished by simply checking to determine whether the radiation frequency is covered by ESM gear within the radar horizon. Some existing ESM equipment may fail to detect because of faulty maintenance; or it can have significant time delay before detection occurs. Although a model of ESM detection probability vs. time might be useful in dealing with this equipment, it does not appear to be a high priority issue for the time period of interest.

4.2 LOCATION

ESM systems can also play an important role in the development of location information, but at least at present, the enemy is more likely to rely on his own radar to obtain location information on individual U.S. units. Of course, if for some reason he is attempting to operate in a completely passive mode, he may try to obtain location information by triangulation methods, using the direction-finding capabilities of his ESM equipment.

Such triangulation can be accomplished with a single airborne sensor by using the direction-finding capability from different points on the aircraft trajectory. However, the angular accuracy limitation of present ESM direction-finding equipment together with the operational problems of plotting and coordination tends to limit the usefulness of triangulation.

In the future, a much more accurate passive location capability can be provided by analyzing the time of arrival of radar pulses at three or more ESM receiver locations. Although, in theory, this type of position finding can be accomplished very accurately without a direction-finding capability, it does require good communication channels between receivers that are separated by considerable distances, and this can be a serious problem for an opponent that is trying to operate in a completely passive mode.

Because of the coordination and communication problems involved in the passive location methods, it seems likely that, for the near future, military forces will prefer to rely on active radar surveillance for position information, since airborne surveillance radars can be used without giving away any significant information about the location of high-value surface units.

4.3 IDENTIFICATION

One serious limitation of such active radar surveillance is that it does not provide information on the identity of the surface units that are being observed. Thus, it is necessary to supplement the radar data by other sources in order to obtain the identification information. In periods of good visibility, visual air surveillance is usually the most reliable source. But at night and during periods of poor visibility, visual identification may not be possible. Moreover, there can be tactical reasons why air surveillance cannot be used. Thus the identification function is probably the most

important combat role for ESM equipment. It is a function that is needed regardless of whether the position information is provided by active radars or by passive monitoring methods. When ESM gear is used in this way, the direction-finding capability plays an essential role because it makes it possible to associate passive emission signals with particular radar returns. Under such situations, ESM becomes an essential link in the identification process. One of the most important functions of EMCON plan is to make sure that this ESM identification function will not be easy for the enemy.

To put the identification problem into an operational perspective, it is helpful to visualize the environment of a combat information center where all participants in a battle or a crisis situation appear only as spots on a radar display. Some spots are brighter because they correspond to larger ships, but there is little or no information designating which ship is which. Some of the spots actually correspond to commercial shipping, some to neutral, friendly, or hostile warships. Some of the hostile warships, such as destroyers, may be of relatively minor importance compared to others that may be cruisers or aircraft carriers. In order to know what targets to shoot at, how to allocate firepower, and how to maneuver for combat advantage, it is vitally important to know which ship is which.

The ease with which the enemy may be able to manage the identification function will depend to some extent on the quality of his communications between ships. If he has operational data links, such as those provided with the NTDS, he may be able to communicate track and identification information between ships. However, if he is operating under EMCON communication rules that severely restrict such communication, each ship may be largely on its own with regard to the identification function. In either case, however, it can be helpful in planning EMCON procedures to have a way of assessing the amount of identification information that is provided by one's own emissions.

The proposed emissions analysis decision aid takes the form of a Bayesian inference analysis system, which evaluates the effects of the electronic emissions on the opponent's ability to estimate which ship is which. If there is no identification information, then each radar return (or observed emitting platform) is equally likely to be associated with any of the ships in the area. If we credit the enemy with an ability to use the strength and size of the radar return, then he may be able to classify ships to some extent by size, but he still would not be able to make any distinctions within a size group. 1 Through the use of ESM, he may be able to refine such general estimates. He may be able to definitively identify certain ships as U.S. Navy ships. Or, he may be able to uniquely identify a ship as a U.S. aircraft carrier, or even a specific aircraft carrier. The Bayesian analysis system can provide a quantitative measure of the degree to which enemy ESM efforts can reduce his uncertainty about the identity of each U.S. ship. Such a system can evaluate the degree of uncertainty concerning the identity of each blip that would remain after an analysis of the ships' electronic emissions. Within the system, this estimate takes the form of an identification probability matrix P_{ij} , which specifies the opponent's Bayesian estimate of the probability that the radar return, i, might actually be generated by ship Since each ship must be somewhere, the summation of probability for any ship (or any radar blip) should always be equal to 1.0. Specifically,

$$\sum_{j} P_{ij} = \sum_{i} P_{ij} = 1.0$$

If the identity of blip i is reliably known to be ship k, then $P_{ij} = 1.0$ if j = k; and $P_{ij} = 0$ if $j \neq k$. This matrix representation, of course, provides a complete description of the uncertainty in identity for each observed platform. From a theoretical perspective, the calculation of the P_{ij} matrix is a simple application of Bayesian inference mathematics.

Such radar "size" information is vulnerable, of course, to the use of deceptive repeater jammers that can increase the apparent size of a radar target.

In practice, however, the problem is not so simple because the required calculation increases very rapidly as the number of ships increases. Consequently, to provide a practical computational system, it is necessary to use approximation methods. See Sec. A.1 of the appendix for more details.

Although the system is described in a context where the opponent is using active radar to provide location information, it is just as applicable to any situation where he can associate some combination of emitters of unknown identity. For example, in cases where the platforms themselves are identified only by passive methods, the same mathematical methods would be needed to analyze the identification problem.

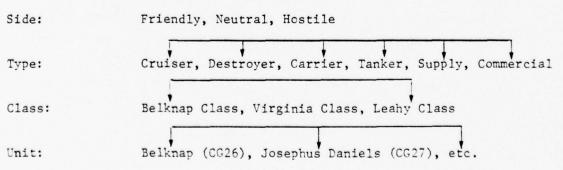
Because of the completeness of the information contained in the P_{ij} matrix, the matrix can be used to answer any specific user queries about the resulting formation status. Undoubtedly a number of standard formats should be provided to anser questions such as: What other ships, or types of ships, are likely to be confused with any high-value ship of interest? What are the probabilities of such confusion? Similarly, the same basic probability matrix can be used to define the information state so that it can be analyzed or evaluated by other computerized algorithms.

However, if the user is dealing with a large Task Force, the size of the basic matrix can become very large, and it may become very tedious to examine different EMCON alternatives in terms of such detailed information. Therefore, to facilitate the examination of a larger number of alternatives, the system design allows the user to define a single qualitative "score" that he can use to make a first-cut comparison between different EMCON alternatives. Since the specific issues of importance can vary widely depending on the specific objectives and environment of the task force, the scoring criteria are designed to make use of value judgments that can be supplied by the user. Two different scoring procedures have been designed. One is concerned with errors the opponent is likely to make

in assessing the value of ships as potential targets. The other is concerned with the errors he is likely to make in a more detailed classification of the ships by type and class.

It is clear that the importance of identification information depends on the value of the ship that is being identified. It is usually more important to know the identity of an aircraft carrier than of a destroyer, so this difference in the value of what is being identified should be reflected in the scoring procedure. If we look at the problem from the perspective of the enemy's capability to target our force, it becomes apparent that what is most important is the enemy's ability to estimate the value of each potential target. For example, if a task force contains two carriers, we have very little to gain by confusing the enemy in distinguishing between them. But, if we can leave him confused between a low-value unit like a destroyer and a high-value unit like a carrier, there is a very real loss in his ability to do effective targeting. Thus, one useful measure of the quality of identification information available to the enemy is provided within the system by evaluating the extent to which the enemy's estimate of the value of the units as potential targets is uncertain or distorted.

But there are also qualitative as well as valuative factors that are important in assessing identity information. For example, a tanker and a destroyer might be of essentially equal value, but they pose very different threats, and the appropriate tactics for dealing with them should be quite different. It is important to be able to recognize ships by class and type even though the ships may be of very similar value. The structure of the available classification information can be usefully approximated in a hierarchial form. The categories might be approximated as follows:



The minimum level of knowledge would be side only. If the side is correctly identified, the additional credit would be given for knowing the type; if the type is correct, additional credit would be given for knowing the class; and, finally, if type and class are correct, additional credit would be given for knowing the specific unit.

To allow for both the <u>value-oriented</u> and the <u>classification-oriented</u> points of view, the decision aid is designed so that the user can decide how much weight he wishes to attach to each of the two scoring methods. Thus, the user could decide to place 40% weight on the value point of view, and 60% weight on the classification perspective. In addition, he might decide that, with regard to the classification perspective, he would assign 30% for side, 40% for type, 20% for class, and 10% for unit information. Finally, he could make his own estimate of the value for each unit involved. With this definition of the decision maker's value judgments, the system can evaluate, on a scale from zero to 100%, the amount of identification information that is available to enemy ESM equipment. The score

 $S = \alpha V + (1 - \alpha)C$

would be given as weighted sum of two components, the value component V and the classification component C. The user would select the weighting coefficient α to reflect the importance he wishes to assign to value—oriented as opposed to classification—oriented scoring.

To estimate the score V in the value component, we define V as a dot product between two unit vectors, where one vector is proportional to the actual value \mathbf{v}_i of each ship in the task force as estimated by the decision maker, and the second vector is proportional to the best Bayesian estimate $\tilde{\mathbf{v}}_i$ of the value that is available to the opponent.

Using the probability matrix $\mathbf{P}_{\mathbf{i}\,\mathbf{j}}$ to estimate $\tilde{\mathbf{v}}_{\mathbf{i}}$, we obtain

$$\tilde{v}_{i} = \sum_{j} P_{ij} v_{j}$$

Normalizing both vectors to unit vector size, we obtain a value $\mbox{\bf V}$ for the dot product given by

$$v = \frac{\sum_{i} v_{i} \tilde{v}_{i}}{\sum_{i} v_{i}^{2} \times \sum_{i} \tilde{v}_{i}^{2}}$$

Of course, the opponent could be confused not only about which ships are which within the Task Force, but also between the elements of the Task Force and other commercial and neutral or even the opponent's own ships. To provide a proper reflection of such confusion with non-task force elements, the values of commercial and neutral shipping should probably be set to zero (or at least very low) and the values of the opponent's own ships should be treated as negative, to reflect the seriousness to the opponent of confusing friendly and hostile elements as potential targets.

To estimate the value of the classification component C, we once again make use of the P probability matrix. But we define a new parameter $\delta_{\mbox{ijk}}$ which is indexed over the hierarchical categories, k:

k = 1 for side

k = 2 for type

k = 3 for class

k = 4 for unit

By definition δ_{ijk} is equal to zero unless ship i and ship k fit in the same classification at all levels of the hierarchy up to and including the kth level. Using this definition of δ_{ijk} , we can define a similarity score γ_{ij} which reflects quantitatively the similarity of two ships i and j by category. Specifically, we define

$$\gamma_{ij} = \sum_{k} \alpha_k \delta_{ijk}$$

where $\boldsymbol{\alpha}_k$ are the weights the planner chooses to attach to each of the hierarchical identification categories.

The total classification information C is then given by

$$C = \sum_{i} \sum_{j} P_{ij} \gamma_{ij} v_{i} / \sum_{i} v_{i}$$

where γ_{ij} is the similarity score.

The preceding formulas define a procedure for reflecting the total amount of identity information that is potentially available to enemy ESM equipment.

The information can be used simply as a scoring procedure for evaluating the information given away by different ESM alternatives, or it can be used in a trade-off analysis with radar detection performance to provide an overall figure of merit for an ESM plan when the trade-off considerations are taken into account.

Obviously, in order to make the trade-off analysis explicit, it is necessary to provide a parallel decision aid that can deal quantitatively with the radar performance issues. The next section develops the concept for the radar analysis decision aid.

5 AN AIR DEFENSE SURVEILLANCE DECISION AID

The ability of a task force to defend against hostile aircraft and cruise missiles depends on the ability to detect such threats at a range sufficient to allow effective interception with other aircraft, missiles, or (as a last resort) guns. The probability of detection depends not only on the physical capabilities of the radar, but also on the alertness of the radar operators. In many operational situations radar operators fail to detect threat aircraft even though the track would be clearly identifiable on the radar screen by anyone who knew where to look. Because of the probability of such human failures, it is usually prudent to provide for some redundancy in the radar coverage. As the EMCON restrictions reduce the number of operational radars, or reduce percentage of time that the radars can operate, there is an inevitable loss in the redundancy of coverage, and a corresponding reduction in the probably effectiveness of the fleet air defense system. This reduction must of course be weighed against the advantages that can be obtained through EMCON procedures that deny information to the enemy. The purpose of the air defense surveillance decision aid is to provide the decision maker with a more quantitative understanding of the effects of EMCON policies on air defense surveillance capabilities.

As in the case of the EMCON identification issue, the planner needs a number of different representations of the problem corresponding to different levels of detail in his consideration of the problem. At the most detailed level the planner would undoubtedly like to see a geographic plot of his surveillance coverage for any EMCON option he wishes to consider in detail. Thus, the surveillance decision aid includes a capability to produce contour plots of radar coverage.

In the past, plots of radar coverage and effectiveness have been calculated in several different ways. The most common plots have displayed simply a "maximum detection range" with perhaps some other contours corresponding to different cumulative probabilities of detection for each radar.

Unfortunately, these traditional contour plots depend on the assumption that the incoming aircraft is flying directly toward the radar and that the radar has been on continuously. Such plots do not provide quantitative information on the combined effectiveness of multiple surveillance radars and they do not provide valid information if the threat aircraft has changed direction, or if the radars are to be switched off and on periodically as has been suggested in some of the more recent EMCON plans.

To provide plots that will correctly reflect surveillance effectiveness in such situations, the design includes an alternative display showing contours of equal detection probability per unit time. Such contours have the advantage that they can be easily combined to reflect the effects of redundancy in radars and operators. The resulting contours can be labeled so that they should have a natural intuitive meaning since each contour also corresponds to a fixed expected time delay before a previously undetected object is likely to be detected. If the time delay is of the order of 1 or 2 seconds, the probability of prompt detection will be very high. However, in an area where the expected time delay is around 60 seconds there is a high probability that an aircraft will be able to penetrate without being detected. The usefulness of such a display to the planner, will, of course, depend on the ease with which the planner can become comfortable with this new type of display. Tests should be conducted to determine its practicality.

Ideally one might like the display to reflect the effects of terrain and weather factors. It would not be difficult to include the effects of sea state and even of a general rain environment on detection probabilities, and this should probably be included in an operational system. However, it would be much more complex to include the effects of local terrain features (mountains, islands, etc.) and isolated weather features such as clouds and squalls. Because of the data input problems of dealing quantitatively with such local features it is probably more realistic to leave these factors to human judgment even in an operational system. In the

present prototype system the effects of weather and terrain are simply treated for an average or typical clear sea situation.

With this limitation, the decision aid provides a contour map of radar detection probabilities over the entire Task Force, together with standard detection range contours. To use the decision aid the user will specify nominal positions for the radar platforms (ships and CAP aircraft) in the Task Force. He will also want to supply his own estimate of the proficiency of his radar operators. The contour plot will then display the resulting contours in relation to the configuration of the Task Force. A given plot will, or course, depend on the specific hostile air threat that is specified, and it is a function of the radar cross section and penetration altitude of the hostile aircraft. Thus, the user's various plots will correspond to differing threat assumptions.

This kind of detailed representation is necessary not only to allow the user to study specific alternatives in detail, but also to allow him to develop an intuitive understanding of the analysis system. However, such a detailed display actually provides too much information to be very easy to use when many alternatives must be considered. Consequently a more compact way of representing the results is required, such as an overall "figure of merit" that can be calculated for each alternative. The following paragraphs discuss a procedure that the planner can use to obtain such a figure of merit for each radar surveillance plan.

Since the main purpose of the air surveillance plan is to protect the units of the Task Force and the importance of providing the protection is highest for the high-value units, it seems appropriate to define the surveillance figure of merit, S, as a summation over the units of the Task Force, in which each unit i is weighted in proportion to its value v;:

$$S = \sum_{i} v_{i} \phi_{i} / \sum_{i} v_{i}$$

where v_i is the value of the $i\underline{th}$ unit and ϕ_i is a measure of the quality of surveillance protection provided for the value unit.

To give the surveillance score an intuitively significant meaning, ϕ , for each individual unit should be defined in terms of simple and intuitively significant outcomes. This could be done in various ways. For example, (1) ϕ_i might represent the percentage of penetrators expected to get within strike range of the unit before being detected; or (2) it might represent the number of hits that the penetrator might be expected to achieve on the unit; or (3) it might represent the expected percentage damage against the unit. Each of these alternatives has certain advantages and certain disadvantages. The first has the apparent advantage of simplicity since it can be estimated without dealing with factors such as the hit probability per penetrator or the kill probability per hit, which go beyond the issue of surveillance. In particular, if the user can define what he means by "strike range," then the probability that an aircraft will penetrate within strike range without being detected can be calculated directly from a radar detection probability model. Thus the first measure avoids involvment in other detailed interactions that go beyond the issue of surveillance quality.

On the other hand, it is difficult to assess the military significance of such a measure. What is the meaning of "strike range," and how serious is it if some estimated percentage of the hostiles penetrate inside this range without being detected? How does the importance depend on the value and vulnerability of the unit in question?

The use of the expected number of "hits" as suggested in the second measure provides a more concrete and intuitive output for which it is easier to judge the military significance. However, if we were to use the estimated number of hits in a computer program as a direct measure of surveillance protection for each unit, it would be equivalent to assuming that the importance of a hit is the same regardless of the type of unit that is involved. Actually of course, the importance of a hit

depends both on the value and vulnerability of the unit that is hit. Consequently, in order to put the importance of the hits on a comparable scale for all units, it is necessary to take these additional factors into account. The third alternative accomplishes this objective by converting the estimated number of hits into an estimated percentage damage for each unit. When the percentage of damage is weighted by the value of each unit, it provides a way of placing the surveillance score for different units on a roughly comparable basis.

The disadvantage of the last two alternatives is that they require an estimate of factors such as hit probability and percentage damage per hit that go beyond the issue of simple surveillance performance. Nevertheless, it is only when these factors are added that it is possible to provide a measure which gives appropriate weight to different aspects of the surveillance problem.

As a result of the preceding considerations, the following general concepts were developed for defining the surveillance quality measure ϕ_i .

- It would be inappropriate to try to provide a detailed combat model to calculate outcomes such as expected number of hits and percentage damage which involve detailed considerations outside the surveillance area. Therefore, the conversion factors (number of hits and expected damage) that are needed to convert from pure surveillance performance to a properly weighted surveillance score will be based simply on user-supplied planning factors.
- 2. The system will be designed so that the user will have access to any intermediate factors that go into the calculation of the surveillance quality score ϕ_i for each unit i and will be able to base his judgment on these intermediate factors as well as the overall score. Thus he will be able to use the basic system even if he is not inclined to provide the planning factors that are needed for some of the more sophisticated planning aids.

The quality score ϕ_i is calculated from user-supplied planning estimates and briefly discussed below. For more detail, see the appendix (Sec. A.2).

Let $h_i(r)$ be a function that reflects the planner's estimate of the number of hits that can be expected against unit i if a hostile attacker is first detected at range r. Let $P_i(r)$ be the probability (as calculated from the surveillance model) that the threat would first be detected at range r. The expected number of hits H_i on the unit can then be calculated as follows:

$$H_{i} = \int_{0}^{\infty} h_{i}(r) P_{i}(r) dr + h_{i}(0) \left[1.0 - \int_{0}^{\infty} P_{i}(r) dr \right]$$

The first term gives the total expected number of hits for penetrators detected before they reach r=0, and the second term gives the expected number of hits for penetrators that are not detected before r=0.

For analysis purposes it is probably appropriate to use a single function $d_i(h)$ which reflects the planner's estimate of the expected percentage of damage against unit i as a function of the number of hits h. Since the number of hits $h_i(r)$ is estimated as a function of the detection range r, one can easily calculate a range-dependent expected damage

$$d_{i}(r) = d_{i}[h_{i}(r)]$$

in which the expected damage against the \underline{i} th unit is expressed simply as a function of the detection range. This damage function can be used to calculate the total expected damage just as the hit function $h_{\underline{i}}(r)$ was used to calculate the total number of hits. Specifically the total expected damage $D_{\underline{i}}$ is given by

$$D_{i} = \int_{0}^{\infty} d_{i}(r) P_{i}(r) dr + d_{i}(0) \left[1.0 - \int_{0}^{\infty} P_{i}(r) dr\right]$$

With both options available the user can obtain information either on the expected number of hits, or on the expected percentage of damage. The damage criterion, however, is the only measure appropriate for use as the basis for an overall surveillance score, S. Only this damage criterion satisfies the requirement of providing a score ϕ_i , which, when multiplied by target value V_i in the formula for S, can correctly reflect the differences in value and vulnerability for the different units in the task force so that different surveillance plans can be appropriately compared. Therefore, in the material that follows we will identify the surveillance effectiveness ϕ_i with the probability of damage D_i .

In discussing the surveillance quality measure ϕ_i , we have so far acted as if the probability of penetration to a given radius is well defined for a given radar configuration. Actually, of course, it depends upon the direction of approach used by the penetrators. For each penetration azimuth, the probability of penetration to a given range will be different and the resulting expected damage D_i and the expected number of hits H_i will also be different. Thus, we can add an index j for the different possible attack azimuths and for each azimuth j we can calculate the expected number of hits H_i and the expected percentage damage D_i .

Since enemy attacks are likely to be designed to take advantage of defense weaknesses, our measure $\boldsymbol{\varphi}_i$ of the overall surveillance effectiveness at target i should be most sensitive to the effectiveness along the weakest defense axis. If we were sure that an attack against unit i would come from the direction of least surveillance coverage, we could define the effectiveness $\boldsymbol{\varphi}_i$ as the probability of defense success against such a threat:

$$\phi_{i} = \max_{j} (D_{ij})$$

However, such a measure would be completely insensitive to the surveillance coverage against other threat directions. We can provide an improved measure that takes all directions into account by defining $\boldsymbol{\varphi}_{\boldsymbol{i}}$ as follows:

$$\phi_{i} = \left(\sum_{j} \frac{1}{n} D_{ij}^{\gamma}\right)^{1/\gamma}$$

This function has the desirable property that an improvement in defense effectiveness along any threat axis will always yield an improvement in the measure of merit ϕ_i . For $\gamma=1.0$, it provides a simple equal weighting of all threat directions. However, in the limit as γ approaches infinity it provides a result which approaches the effectiveness along the weakest defense axis. For intermediate values of γ such as $\gamma=2$ or $\gamma=3$, it should produce intermediate results which place an appropriate increased emphasis on that threat axis for which the surveillance coverage is least effective.

If there were only one type of air threat we could provide an overall measure S of surveillance effectiveness simply by applying the formula given earlier:

$$S = \sum_{i} v_{i} \phi_{i} / \sum_{i} v_{i}$$

However, this is actually a somewhat oversimplified representation since the planner will usually want to consider several different threats corresponding to different radar cross sections, altitudes, and penetration speeds. To provide an overall measure covering such a mixture of threats the user will probably want to specify a mixture of threats each with its own characteristics and provide his own judgmental weighting of the likelihood of the various threats. Thus for each threat k, there would be an

estimated weight or importance W_k , and a calculated surveillance effectiveness S_k . The overall surveillance effectiveness \overline{S} would then be calculated as follows:

$$\bar{S} = \sum_{k} W_{k} S_{k}$$

This information can be provided in a number of alternative ways. For example, on the geographic or map-type display the overall surveillance score could be given on the side, as illustrated in Fig. 2 (see Sec. 6). Individual surveillance scores for the individual units could also be displayed beside the units (instead of the ID numbers now shown in Fig. 2).

The information could also be displayed in a matrix from which could give a detailed analysis by threat and by individual Task Force elements. A single entry in the matrix would display the surveillance score for a particular unit, i, relative to a specific threat k. The column averages would give the overall value weighted surveillance score averaged over all units for each threat, whereas the raw averages would give the overall surveillance score for each unit based on a weighted average of the threats.

6 COMBINED AIR DEFENSE/EW DECISION AID

If both the air defense surveillance decision aid and the emissions analysis decision aid are available, it is a rather simple matter to explicitly produce from them a combined measure of merit or score. If \mathbf{S}_1 is the surveillance score and \mathbf{S}_2 is the score for emission information given away, we can define a total combined score $\mathbf{S}_{\mathbf{T}}$ as follows:

$$S_T = (1 - \beta)S_1 + \beta(1.0 - S_2)$$

The quantity β is a user-chosen weighting factor which reflects the fraction of the weight he wishes to attach to the emission information issue. The information score, S_2 , is introduced with a negative sign because it is defined in terms of information given away. The best information score, therefore, occurs when S_2 = 0, whereas the surveillance score was defined so that the best score occurs for S_1 = 1.0. The equation above corrects for this difference in definition so that an improvement in either score will produce an increase in the combined figure of merit S_T .

With such a combined figure of merit, it is possible to provide scores that should assist the planner in choosing between alternatives. Moreover, the availability of such a combined measure makes it possible to provide computer-assisted optimization methods, if desired.

We will consider first how the scores might be presented to assist the planner in making his decision. Figure 2 illustrates a kind of display that should be useful. The table at the top of the display gives the raw surveillance and EW INFO Scores for each alternative. The graph below shows the combined scores for each of the alternatives as a function of the weight the planner chooses to assign to the two objectives. In the example shown, alternative C completely dominates alternative B for

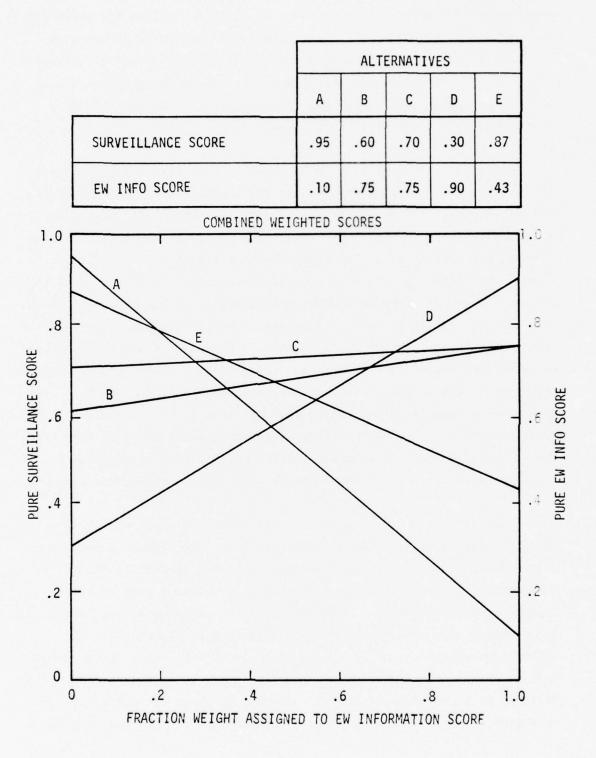


Figure 2. Display of Combined Surveillance and EW INFO Scores

all values of the weights. The choice between the other four alternatives depends on the user's judgment about the appropriate weights. The top envelope of line segments in the order A, E, C, D, defines the preferred strategies as a function of the importance attached to EW information. The planner can use this type of a display to help in selecting alternatives. If he were to decide that plan E was his preferred alternative, this would suggest that he was mentally assigning a weight of about 0.3 to the INFO score, for it is only in this range that plan E appears superior.

Up to this point we have been concerned only with the calculation and evaluation of the outcomes for any EMCON plan the planner might wish to try. However, if the planner can specify an appropriate weight β that he wishes to assign to the INFO score, then it is theoretically possible to provide a computerized search algorithm that will help the planner to improve his plans. Although no such search algorithm is included in the present design, it might be a worthwhile addition to the design assuming that the rest of the decision aid proves to be satisfactory. In the case just mentioned the planner might be able to use such an algorithm to develop an improvement over plan E that would yield a better score using the weight 0.3. The nature of the problem, however, is such that there are likely to be numerous alternatives that are locally but not globally optimum. Consequently, although computerized search procedures might be very effective in improving on a basic EMCON concept provided by the planner, it probably would not be very effective in developing entirely new concepts.

Thus, if the search capability were added, we could visualize a method of operation somewhat as follows. After the planner defines some of his basic alternatives and obtains a plot such as that in Fig. 2, he then makes a judgment about the importance or weight β that he wishes to assign to the EW INFO Issue. He then asks the computer to test variations in the plans that appear best for the weighting he has chosen. The computer proceeds to examine a long sequence of variations in the neighborhood of the planner's original choice. As a result of this search, it delivers an improved plan that yields a higher score for the specified weighting factor. The planner could then examine the "improved" plan to

see if he believes it is really an improvement over his original concept. The new alternative might be added to the display as a new alternative F, and a new line corresponding to that alternative would appear. In the case just considered, if the computer is successful it should produce a new alternative F that would be above alternative E at least in the region close to weight 0.3.

7 IMPLEMENTATION OF PROTOTYPE

As part of the present contract, work began on the implementation of the prototype system. The effort was divided into two parts: the detailed design and implementation of the air surveillance display and analysis subsystem, and the completion of some mathematical research required for the information analysis subsystem. An initial version of the air surveillance subsystem was completed and demonstrated at the ODA contractors meeting on 16 December 1976. This section discusses the system that was demonstrated and recommends some improvements that have developed as a result of comments and suggestions at the demonstration.

7.1 INTERFACING WITH THE DATA BASE

An emission control planning system must necessarily make use of a large amount of technical data. The necessary data (in unclassified form) has been collected by CTEC and stored in the University of Pennsylvania relational data base. In order to utilize this data as it is required within the decision aid, procedures were developed so that the decision aid itself could automatically generate the necessary data retrieval commands for the relational data base.

The GRC prototype system was the first externally developed system to exploit these potential capabilities of the University of Pennsylvania data management system. The few system bugs that were encountered were quickly corrected and the process was surprisingly smooth. It proved quite simple to utilize the available commands to exploit the relational characteristics of the data base.

7.2 AN INTERACTIVE COMMAND LANGUAGE

In order to allow a user to work efficiently with a decision aiding system, the system itself must be provided with a flexible command language. (which is separate from the basic data management system language, DAISY) needed for a wide variety of purposes:

- To inform the analysis system what specific ships and aircraft are relevant for a particular analysis so that the relevant data can be retrieved from the overall data base.
- 2. To give the user flexible control over the positions of ships, and the specific equipments that are assumed to be turned on.
- 3. To provide the user with a general-purpose inquiry language that allows him to obtain answers to a wide variety of questions he may wish to ask concerning the status of forces he is working with and intermediate computational results that may be of interest.
- 4. To allow him to invoke specific standard data formats and graphical displays when he wishes to do so.

Although superficially it might appear that the interactive language provided by the University of Pennsylvania in its DAISY system should be applicable for this purpose, a closer examination shows that this is not so. The DAISY language is designed to operate on the basic relational data base. Once information has been retrieved from this data base and modified for use by a specific decision aid, it has been converted to a new format specific to the particular decision aid. Thus, the user must be provided with a separate command language which allows him to interact with and efficiently control the decision aid. The command language that was developed for the surveillance subsystem should be easily expandable to control all the components of a more complete emissions control decision aid.

7.3 THE AIR SURVEILLANCE DISPLAY

Figure 3 shows a reproduction of one of the video displays provided by the air surveillance component as it was demonstrated at the 16 December meeting. The system as demonstrated included only what we are not describing as the alternative form of the surveillance display. Specifically, it was a display of surveillance contours showing the mean time to detection for an undetected threat at any position relative to the Task Force.

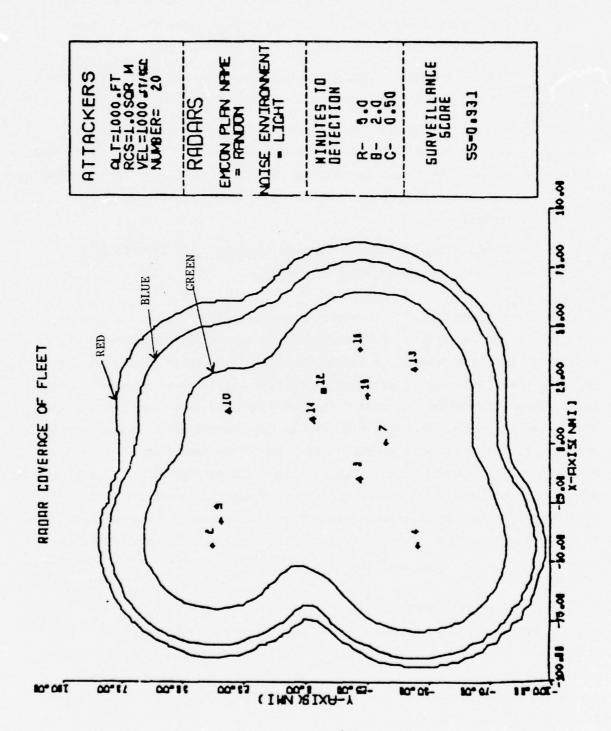


Figure 3. Display of Air Surveillance Converage of Fleet

Though this reproduction of the display is in black and white, the actual display was in color. The left side of the display provides an X-Y plot showing the location of the various units of the fleet, together with a contour plot of the radar surveillance coverage. Three different contours are shown. The green inner contour surrounds a region of good quality radar coverage where the mean detection time for the assumed threat should be 30 sec or less. The blue contour corresponds to an intermediate region where the mean detection time should be about 2 min. The red contour marks the effective limit of radar coverage where the expected detection time should be 5 min or more.

The surveillance contours that are shown are not round because they reflect the accumulated detection probabilities for all of the surveillance radars in the fleet that have been specified as "on." The quality of surveillance shown at each point represents the accumulated detection probabilities of all radars covering that point. This kind of representation was expected to be useful in assessing the quality of the coverage since it specifically displays the advantages of redundant coverage.

Comments during the demonstration, however, made it quite clear that this form of display was unfamiliar and that Naval officers would not relate to it easily at least without a careful explanation of its meaning. For this reason, the decision-aid design has been modified to include a more traditional alternative which simply displays the circular coverage region for each radar.

The legend at the right of the display shows the characteristics of the assumed threat: altitude, radar cross section, velocity, and number of attackers. It also gives some information about the assumed radar environment, the meaning of the surveillance contours displayed, and the overall surveillance score which might correspond to the expected survival probability of the unit that is assumed to be under attack (weighted, of course, by unit value).

8 WEIGHTING AIR SURVEILLANCE VERSUS INFORMATION GIVEN AWAY

In the original design as it was presented at the meeting, it was assumed that the user would apply his own judgment to assess the proper weight that should be given to the information obtained by active surveillance as opposed to the information given away to the opponent by one's own emitters. Comments on this approach made it quite clear that there might be a need for a more objective analytic procedure to assist the user in making this type of judgment.

In order to provide an objective basis of comparison, it is, of course, necessary to define rather concretely the way the enemy might capitalize on information given away. Since there are many different types of scenarios that could occur, it is not possible to define a single universal procedure that could be used to make the comparison. Nevertheless, for many applications the following approach seems to provide a useful basis of comparison.

Suppose we are concerned with the risk of a surprise air attack against the fleet. Moreover, assume that the attack could be accomplished by over-the-horizon targeting of cruise missiles, or by an aircraft attack in circumstances of low visibility so that the opponent would be dependent on electronic methods to identify potential targets. Under these circumstances, there is a rather well defined trade-off between information given away and the quality of the air surveillance.

For any given enemy attack, the air-defense effectiveness will tend to increase as more surveillance radars are turned on; and maximum effectiveness will usually be reached when all radars are in operation. On the other hand, as more radars are placed in operation, more information will tend to be given away to the enemy concerning the identity of the ships in the Task Force. Obviously the more information the enemy has, the more accurately and efficiently he can plan his attack. The optimum EMCON plan therefore should be one which keeps enough radars in operation to provide an effective air defense, but which avoids giving away much identity information that would help the opponent in planning an efficient attack.

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To provide an analysis of the trade-off between information given away and the effectiveness of air surveillance, we can analytically simulate both the planning and execution of such an attack.

Specifically, for any emission control plan we can utilize the information analysis subsystem to evaluate what information is potentially available to the enemy about the identity of our ships. On the basis of that incomplete information, the analysis system can project an enemy strike plan in which the available strike resources are allocated as efficiently as possible to the units of the fleet within the limits of available information. The success of the strike can then be evaluated, on the basis of the quality of the surveillance coverage of each ship in the fleet, to obtain an estimate of the expected damage from the attack.

This technique should provide a useful quantitative comparison. Obviously it is not the whole story since the commander will undoubtedly wish to consider many other factors. Nevertheless, it should provide a useful quantitative comparison which could serve as a starting point for his consideration of the other factors. The design concept has therefore been extended to include such a quantitative comparison capability.

9 CONCLUSION

The purpose of this document is to provide a status report on the present state of a research and development project. Considerable work remains to be done to complete the decision aid and assess the usefulness of the concepts. For this reason, no attempt has been made to present the results in a form suitable for a final report of a research project.

Obviously, when the decision aid is completed detailed documentation from both a user and a programmer perspective will be necessary. Inclusion of such material in the present report would be premature. The objectives of the present report are much more limited: to explain the objectives of the decision aid, to provide an introduction to the design concept, and to provide a report on the present status of the development.

APPENDIX

MATHEMATICAL ANNEX

This annex is divided into two sections. Section A.1 is concerned with approximate methods for evaluating Bayesian probabilities within the information analysis subsystem. Section A.2 is concerned with the mathematical methods used to evaluate surveillance performance.

A.1 APPROXIMATE EVALUATION OF BAYESIAN PROBABILITY

From a mathematical point of view the information analysis subsystem is concerned with the calculation of a single matrix, \mathbf{p}_{ij} , which represents the identity information available to the opponent as a consequence of the information provided in our own ship's emissions. Specifically each element in the matrix shows the opponent's estimate of the probability that a specific observed platform, i, actually corresponds to a specific ship, j.

To develop this p_{ij} matrix the algorithm begins with another matrix, q_{ij} . The q_{ij} matrix represents a simple heuristic estimate of the likelihood that ship j would or could produce the combination of radiations observable from platform i. The initial calculation of the q_{ij} matrix is accomplished very routinely by a simple processor which compares the emission capabilities of each ship j with the actual emissions of platform i. If the radiations from the platform i are feasible, or probable, for ship j, then q_{ij} is set rather close to 1.0. If ship j lacks the electronic gear to produce the observable radiation at platform i, then the value of q_{ij} is set close to zero.

The basic mathematical problem is to use this q_{ij} matrix as input data to an algorithm in order to generate the probability matrix p_{ij} . Although the p_{ij} matrix is related to the q_{ij} matrix, the two matrices are actually quite different. The q matrix is independently estimated for each ship/platform correspondence. The p_{ij} matrix, on the other hand, recognizes that each platform has to be explained in terms of the emissions of some ship, and that each ship must be represented in the emissions of some platform.

The standard Bayesian approach to the problem is very simple—at least in principle. In theory we simply consider all possible ways the n ships could be identified with the n platforms, and for each of these possible "states" we calculate an overall likelihood.

A specific "state" is defined by a specific association of each ship with one of the n platforms. The overall probability for such a "state" is calculated simply as the product of the q_{ij} for each of the associations in the state. The overall probability p_{ij} that a particular platform i actually corresponds to a particular ship j is obtained simply by summing these calculated probabilities over the subset of possible states for which ship j corresponds with platform i. When this summation of probabilities is divided by the total probability summed over <u>all</u> possible states, it provides a correctly normalized estimate of the probability p_{ij} .

This theoretical method, however, has a very serious practical defect. The number of different ways that n ships can be associated with n platforms is equal to n(n-1)(n-2) ... (3)(2)(1) or n!. If n is equal to ten, then n! is more than three million. If n is larger than ten, n! very rapidly approaches an astronomical number.

To provide a practical decision aid, it is important that answers be obtained quickly, without excessive computational cost, even when the number of ships involved is large. It was, therefore, decided to explore some approximate methods that would work efficiently even when the number of platforms was large.

Two different approaches were undertaken. The first approach involved the implementation of a recursive algorithm which rigorously solved the Bayesian probability problem. However, the algorithm was designed to incorporate some computational shortcuts which it was hoped would make the calculation feasible even for quite large problems.

- The algorithm began by compressing all identical rows and columns into compound rows and columns which could be treated in a single computation (allowing, of course, for the multiplicity of the resulting compound rows and columns). This allowed the grouping of all ships equipped with identical emitters and all platforms radiating with identical emitters.
- 2. The algorithm was designed so that it could then proceed with the full Bayesian calculation to obtain an exact answer. But it was also designed so that it would generally encounter terms in order of decreasing importance. Methods analogous to branch and bound techniques were programmed to allow the testing of approximations which avoid processing terms below a certain critical size.

By adjusting this critical size, statistics were obtained on the accuracy of the results and the amount of calculation required as a function of this cutoff parameter. To provide suitable test problems Monte Carlo statistical techniques were used to generate trial \mathbf{q}_{ij} matrices of various sizes to assess the computational feasibility of the approach.

The exact answers provided by this recursive algorithm were also used to evaluate some very simple heuristic approximations, and analytic research was pursued to try to develop a theoretically justified approximation.

Initially, on intuitive grounds, it was felt that the probability \mathbf{p}_{ij} should be roughly proportional to \mathbf{q}_{ij} , but of course the \mathbf{p}_{ij} are subject to a normalization constraint because in any feasible association of platforms and ships, each platform must be explained in terms of some ship and each ship must be associated with some platform. This gives rise to the normalization requirements that

$$\sum_{j} p_{ij} = 1.0 = \sum_{i} p_{ij}$$

Consequently, a heuristic algorithm was written which began simply with the matrix \mathbf{q}_{ij} and successively multipled rows and columns by whatever factor was needed to normalize each row and column to 1.0. The result was then compared with the correct Bayesian answer. The results were disappointing. Examination of the numbers showed that the heuristic approximation systematically underestimated the actual information available. A quick heuristic fix was tried in which \mathbf{q}_{ij} was replaced by \mathbf{q}_{ij}^2 before the iterative normalization process was used. With this change, results were astonishingly good. In the first cases tried, all values in the \mathbf{p}_{ij} matrix were within about 1% of the correct Bayesian answer! (Subsequent tests, however, showed that errors as large as 3% would sometimes occur.)

Because of these encouraging results, some analytical work was undertaken to either justify the approximation or to develop a more accurate approximation. This work showed: first, that the correct Bayesian answer is invariant to any changes in the q_{ij} matrix that can be accomplished by multiplying rows and/or columns; and second, that when the rows and columns of the q matrix are normalized so that $\sum_{i=1}^{n} q_{ij} = 1.0 = \sum_{j=1}^{n} q_{j}$ then the correct p_{ij} should include a factor of the form $(1-q_{ij})^{-1}$.

It was, therefore, decided to experiment with a generalized form which began with the normalized q_{ij} matrix and estimated a value \tilde{p}_{ij} of p_{ij} as follows:

$$\tilde{p}_{ij} = q_{ij}^{\beta}/(1 - q_{ij})^{\gamma}$$

The resulting \tilde{p}_{ij} matrix was once again heuristically normalized and the results were compared with the exact Bayesian results. This process showed that the best results were usually obtained using $\beta \cong 1.0$ and $\gamma \cong 2.0$. The results obtained in this way typically showed an error less than 1/2 to 1/3 of that obtained with the q_{ij}^2 approximation.

The success of this approximation suggested that one should look for an analytic approximation that might be derived based on the simplifying assumption that only one element of a normalized q_{ij} matrix is known. When this is the case, the expected value of the other terms in the same row or column is given by $(1-q_{ij})/(n-1)$. The expected value of all other terms in the matrix is then given by $(q_{ij}+n-2)/(n-1)^2$.

Given such a matrix one can calculate an exact solution for $\mathbf{p}_{\mbox{ij}}$ which takes the form

$$p_{ij} = \frac{q_{ij}[q_{ij} + (n-2)]}{(1 - q_{ij})^2 + q_{ij}[q_{ij} + (n-2)]}$$

Although it seems likely that this form should produce an even more accurate result than the previous best approximation, the available resources did not permit an experimental test of this approximation. However, it is interesting to note that in the limit of small $\mathbf{q}_{\mathbf{i}\mathbf{i}}$, this new form reduces to

$$P_{ij} = \frac{(n-2)q_{ij}}{(1-q_{ij})^2}$$

which, except for an irrelevant constant factor, (n-2), is identical with the best previous approximation.

Although some additional tests are needed, it now appears that a heuristic approximation based on one of the foregoing heuristic forms is likely to provide a very satisfactory algorithm for the decision aid. The accuracy of the heuristic approximations seem to be entirely adequate (usually better than 1%) and the calculation time is very modest.

The experience with the more rigorous branch and bound methods, in contrast, has been somewhat disappointing. For large problems, the calculation required to obtain comparable accuracy becomes quite high. Inclusion of a very large number of terms often seems necessary to obtain satisfactory results.

Consequently, our current plans are to do a little more research on the heuristic approximations and use one of them as a basis for the final algorithm.

A. 2 CALCULATION OF SURVEILLANCE PERFORMANCE

The basic radar surveillance contour plots produced by the surveillance model provide an estimate of the probability of detection per unit time for a radar target of a given radar cross section, and a given altitude.

The overall probability of detection per unit time is calculated simply by summing the detection probability per unit time for all surveillance radars within range of the target. Thus, the model requires a way of estimating these detection probability rates for each radar as a function of the altitude and radar cross section of a target. The basic probabilities should reflect both the probability of the skin paint showing on the screen, and the probable time delay before an operator will recognize the radar return. A brief review of available models of combined operator and radar performance shows that some careful model development will be required to provide reliable performance estimates. However, the development of such an improved radar surveillance performance model would go well beyond the responsibility of the present ONR project. Consequently, it was decided to make use of a very simple formula which provides reasonable but probably not very accurate estimates of detection probabilities. Before an operational version of the aid could be installed, it would be necessary to develop an improved radar detection model. Moreover in a more sophisticated version of the system, it should be possible to calculate revised contours in the context of various jamming assumptions, but this is not included in the present version of the system design. The material which follows simply assumes that a suitable function exists which allows us to calculate detection probability rates for any position of the threat.

To develop the contour plots, this function is used to calculate the detection rates on a rectangular grid of points, and a standard interpolation system is used to interpolate between these points as needed to develop contours. The contours are developed using an existing contour plotting software package.

The contour plots, however, are used only for display purposes. All of the calculations for the air defense decision aid utilize the original detection rate computational algorithm.

To estimate the expected number of hits, H_{ij} , or the expected damage, D_{ij} , resulting from an attack along a particular threat axis, the following expressions were defined for use in the decision aid:

$$H_{ij} = \int_{0}^{\infty} h_{i}(r) p_{ij}(r) dr + h_{i}(0) \{1.0 - \int_{0}^{\infty} p_{ij}(r) dr\}$$

$$D_{ij} = \int_{0}^{\infty} d_{i}(r) p_{ij}(r) dr + d_{i}(0) \{1.0 - \int_{0}^{\infty} p_{ij}(r) dr\}$$

The previous discussion of the surveillance decision aid, however, did not describe in any detail how the detection rate information would be used to provide an estimate of the probability $\mathbf{p_{ij}}(\mathbf{r})$ of detection vs. range, or how the user estimate of the hit probability $\mathbf{h_i}(\mathbf{r})$ or the damage probability $\mathbf{d_i}(\mathbf{r})$ would be supplied and adjusted to reflect different threat velocities. This annex provides a more detailed treatment of these computational procedures.

Calculation of Hit and Damage Probabilities $h_{i}(r)$ and $d_{i}(r)$

Although in principle the planner could be allowed to input any arbitrary function of r, the definition of such a general function would probably not be the most convenient approach for him. Moreover, the appropriate functions are undoubtedly dependent on the velocity of the aircraft.

To provide a practical way of specifying the function, it will be necessary to use a standard function for which the user specifies only the value of certain key parameters. The following parameters seem to be appropriate:

- ${\bf r}_0$ = the assumed effective range of the weapons on the enemy aircraft (for example, the missile range or the bomb release range). If the penetrator gets within this range, it is already too late to destroy it.
- ${\bf t}_0$ = the minimum defense response time. If the penetrator is detected less than ${\bf t}_0$ seconds before it reaches the range ${\bf r}_0$, it is too late for the defense to take any effective action.
- τ = the probable mean survival time of the penetrator after defense action begins to be effective.
- $h_{\dot{1}}^{\circ}$ = the hit probability against the <u>i</u>th target assuming that the penetrator survives to launch the attack.
- P = the single shot kill probability for such a hit, or the expected damage for one hit.

If these parameters are estimated by the planner and the threat velocity, v, has been given, the computer can estimate the functions $h_{\dot{i}}(r)$ and $d_{\dot{i}}(r)$ as follows:

If
$$r < (r_0 + vt_0)$$
 then $h_i(r) = h_i^{\circ}$

If
$$r \ge (r_0 + vt_0)$$
 then $h_i(r) = h_i^o * exp - [(r - r_0 - vt_0)/vt]$

Although the expression is obviously oversimplified, it takes into account some of the most important planning factors and should give reasonable results. In order to calculate the expected damage for any number of hits, we first calculate the logarithmic kill potential, for a single hit $K_L = -\ln (1 - P_I)$. The expected damage, $d_i(r)$, for n attackers as a function of r is then given by

$$d_{i}(r) = 1 - \exp [n * K_{L} * h_{i}(r)]$$

Calculation of Detection Probability, $p_{ij}^{}(r)$ dr

This function represents an estimate of the probability that a penetrator will first be detected between the ranges r and r+dr. The function must be estimated from basic detection rate information $p_{ij}(r)$ dt, which is displayed in the surveillance contour plot. This basic detection rate information defines the detection probability per second for an undetected penetrator located at range r. For a penetrator approaching at a velocity v, we have

$$p_{ij}(r) dr = Q(r) \frac{1}{v} p_{ij}(r) dr$$

where Q(r) represents the probability that the penetrator remains undetected at range r. To estimate Q(r) we first calculate the cumulative detection probability C(r) up to range r:

$$C(r) = \int_{0}^{\infty} \frac{1}{v} p_{ij}(x) dx$$

If the detection probabilities were really independent in each range interval dx, we could exactly calculate the residual undetected penetration probability Q(r) as follows:

$$Q(r) = \exp [-C(r)]$$

However, in practice this expression will grossly underestimate the probability of a penetrator failing to be detected until it is deep within defense radar coverage. The problem is that the detection probabilities are not independent. If the penetrator has failed to be detected during any one minute, it increases the chance that he will fail to be detected in the next. The operator may be tired, the radar may be incorrectly aligned, the sea clutter may be unusually high, or the operator may simply have his attention focused elsewhere.

Thus for planning purposes the exponential form of Q(r) is really not very satisfactory. What is needed is a function that does not decrease so rapidly with the cumulative function C(r). For the purpose of the present planning aid, the following very simple function appears to be more appropriate:

$$Q(r) = [1 + C(r)]^{-1}$$

The two functions are identical in slope and magnitude in the limit where C(r) = 0.0, but as C(r) becomes large, the revised form of Q(r) declines much more slowly.

Using the revised form of Q(r) the probability $p_{ij}(r)$ dr that the penetrator will be first detected between the image r and r + dr is given by:

$$P_{ij}(r) dr = [Q(r + dr) - Q(r)]$$

$$= \frac{\partial}{\partial r} Q(r) = -\frac{\partial}{\partial r} [1 + C(r)]^{-1}$$

$$= \frac{\partial}{\partial r} Q(r) dr = -\frac{\partial}{\partial r} [C(r)] dr$$

$$P_{ij}(r) dr = \left[\frac{1}{1 + C(r)}\right]^{2} [C(r + dr) - C(r)]$$

Using this form for the quantity $\mathbf{p}_{ij}(\mathbf{r})$ dr makes it quite easy to evaluate the expression in a computer algorithm. The expression for the expected damage that needs to be evaluated is of the form:

$$D_{ij} = 1.0 - \int_0^{\infty} [1 - d_i(r)] p_{ij}(r) dr$$

For evaluation in a computer the expression is written as a summation over increments in Δr . The cumulative function C(r) is then evaluated easily as a running sum if the summation is done in the order from long range to short range. The difference between $C(r + \Delta r)$ and C(r) is, of course, just equal to

$$\frac{1}{v} p_{ij}(r) \Delta r$$

so all parts of the integral are easily evaluated.